Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

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To cite this Article Lin, H. and Palffy-muhoray, P.(1993) 'Non-linear liquid crystal waveguides', Liquid Crystals, 14: 6, 1977 — 1989
To link to this Article: DOI: 10.1080/02678299308027734
URL: http://dx.doi.org/10.1080/02678299308027734

LIQUID CRYSTALS, 1993, VOL. 14, NO. 6, 1977–1989

Non-linear liquid crystal waveguides

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We consider a cylindrical nematic liquid crystal waveguide with an infinite homogeneous isotropic cladding, and study the propagation of TM and TE modes. The dielectric tensor of the liquid crystal core which governs wave propagation is determined by the configuration of the nematic director. For TM modes, propagating optical fields alter the director configuration, and thus change the dielectric tensor. For both TM and TE modes, we consider the effects of the propagating fields on the order parameter tensor. We use an iterative numerical scheme to determine the propagation constant as a function of optical power. For the TM modes, the propagation constant increases continuously with the power. For the TE modes, an abrupt increase is found.

1. Introduction

Liquid crystals are orientationally ordered fluids which possess anisotropic bulk susceptibilities. Due to this anisotropy, liquid crystals experience body torques in the presence of applied fields which may give rise to collective rotation of the constituent molecules. Research on optical field induced reorientation in the nematic phase began around 1980 [1–6]. It is now well known that optical fields can reorient liquid crystal molecules; in most materials, rod-like molecules prefer to align along the electric field. Since the dielectric tensor of non-polar nematics is a function of the square of the applied field, reorientational effects contribute, to lowest order, to the third order non-linear susceptibility $\chi^{(3)}$. In this paper, we discuss how field induced reorientation and changes in the order parameters affect wave propagation in a waveguide, and determine the intensity dependence of the propagation constant for TE and TM modes.

We consider a nematic liquid crystal core confined to a cylinder with radius R, surrounded by an infinite homogeneous isotropic cladding with dielectric constant ε_c. The director is a unit vector in the direction of the local average orientation of the symmetry axes of the molecules. In our model [7], the nematic adopts the 'escape' configuration [8], where the director is parallel to the cylinder axis at the centre, and is perpendicular to it at the surface. The waveguide is shown in figure 1.

In § 2, we study the propagation of the TM mode in the waveguide. The dielectric tensor is determined by the director configuration of the liquid crystal core, which can be obtained by minimizing the Frank free energy. In § 3, we consider the propagation of the TE mode. Here the dielectric tensor is determined by the order parameter tensor of the liquid crystal core, which can be obtained by minimizing the Landau–de Gennes free energy. We consider the biaxial configuration where one eigenvector of the order parameter tensor is fixed in the azimuthal direction.

2. Non-linear effect of TM modes

The dielectric tensor of the liquid crystal core is determined by the director field $\hat{\mathbf{n}}(\mathbf{r})$; $\varepsilon_{\alpha\beta} = \varepsilon_\perp + \Delta\varepsilon n_\alpha n_\beta$ where $\Delta\varepsilon = \varepsilon_\parallel - \varepsilon_\perp$, and $\varepsilon_\parallel$ and $\varepsilon_\perp$ are the dielectric constants for fields

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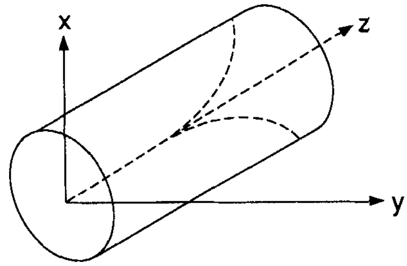


Figure 1. Schematic of the cylindrical waveguide.

parallel and perpendicular to the director. In the presence of optical fields, the Frank free energy density of the system is

$$f = f_{\text{elastic}} + f_{\text{field}}, \tag{1}$$

where the elastic free energy density in cylindrical coordinates (r, ϕ, z) is [7]

$$f_{\text{elastic}} = \tfrac{1}{2}[(K_3 - K_1)\sin^2\theta + K_1]\left(\frac{\partial\theta}{\partial r}\right)^2 + \frac{K_1 \sin 2\theta\partial\theta}{2r\partial r} + \frac{K_1 \sin^2\theta}{2r^2}, \tag{2}$$

where θ is the angle between the nematic director and the symmetry axis of the cylinder and only depends on the radial coordinate r; K_1 and K_3 are the splay and bend elastic constants. The free energy density associated with the optical field is $f_{\text{field}} = -\frac{1}{2}\mathbf{D}\cdot\mathbf{E} - \frac{1}{2}\mathbf{B}\cdot\mathbf{H}$. We assume that the fields have time and position dependence of the form $\exp[i(\omega t - \beta z)]$. Taking the time average and noting that the electric and magnetic contributions are equal, this becomes

$$f_{\text{field}} = -\tfrac{1}{4}(\varepsilon_\perp |E|^2 + \Delta\varepsilon |E_r \sin\theta + E_z \cos\theta|^2), \tag{3}$$

where E_i $(i = r, \phi, z)$ is the magnitude of the ith component of the electric field.

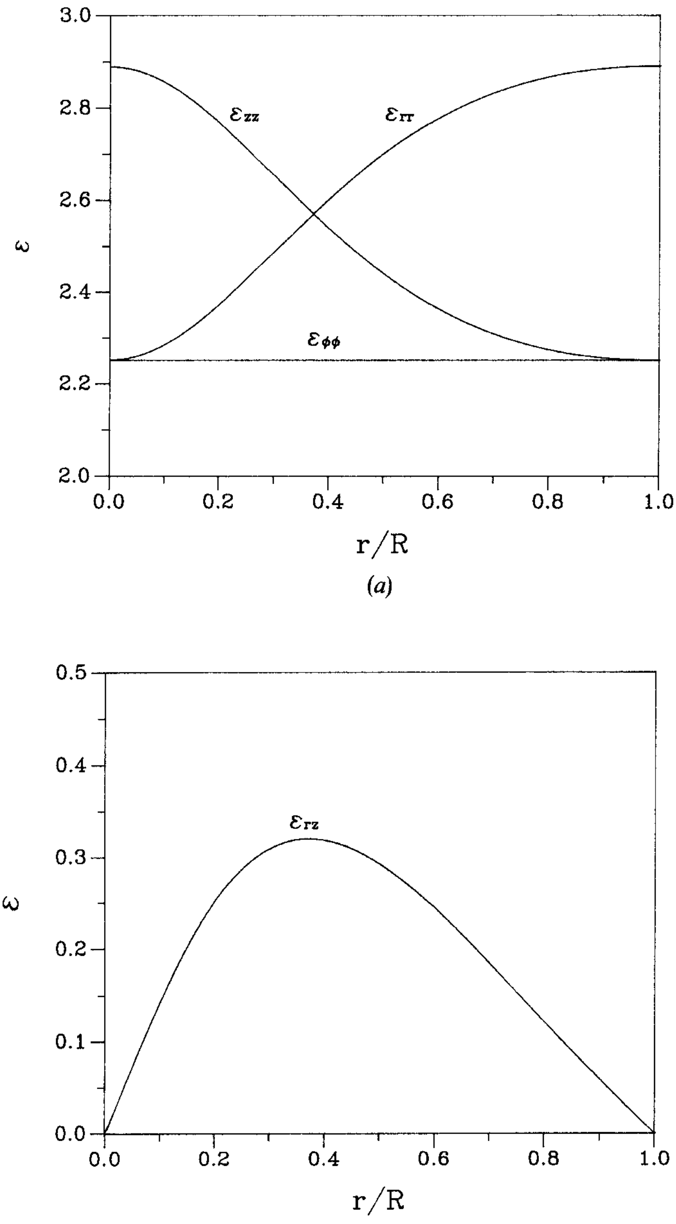
For TM modes, the optical fields have three non-vanishing components [9]: H_ϕ, E_r and E_z. By minimizing the free energy, we obtain the torque balance equation for the director

$$z^2 g(\theta)\frac{\partial^2 2\theta}{\partial z^2} + z^2 \eta \sin 2\theta\left(\frac{\partial 2\theta}{\partial z}\right)^2 + z g(\theta)\frac{\partial 2\theta}{\partial z} - \sin 2\theta + \frac{\Delta\varepsilon R^2}{2K_1} z^2[(|E_r|^2 - |E_z|^2)\sin 2\theta + (E_r E_z^* + E_r^* E_z)\cos 2\theta] = 0, \tag{4}$$

where $z = r/R$, $g(\theta) = 1 + 4\eta \sin^2\theta$, and $\eta = \frac{1}{4}(K_3/K_1 - 1)$. Equation (4) can be rewritten in terms of the time-averaged total propagating power $P = \pi\int E_r H_\phi^* z\, dz$ as

$$2z^2 g(\theta)\frac{\partial^2\theta}{\partial z^2} + z^2(v-1)\sin 2\theta\left(\frac{\partial\theta}{\partial z}\right)^2 + 2z g(\theta)\frac{\partial\theta}{\partial z} - \sin 2\theta + \varepsilon_0 c\left(\frac{P}{P_0}\right)\frac{z^2}{2\pi\int_0^1 E_r H_\phi^* z\, dz}[(|E_r|^2 - |E_z|^2)\sin 2\theta + (E_r E_z^* + E_r^* E_z)\cos 2\theta] = 0, \tag{5}$$

where $P_0 = cK/\Delta\varepsilon$ is a material constant [10], and c is the speed of light in vacuum. $P_0 \sim 5$ mW for 5CB. If E_r and E_z are known, we can solve the equation numerically for $\theta(r)$, and obtain the dielectric tensor in the presence of optical fields. The elements of the dielectric tensor in the absence of fields are shown in figure 2.



(*b*)

Figure 2. (*a*) The elements of ε_{rr}, $\varepsilon_{\phi\phi}$, ε_{zz} of the dielectric tensor. (*b*) The element of ε_{rz} of the dielectric tensor.

The optical fields H_ϕ, E_r and E_z are solutions of Maxwell's equations, and depend on the dielectric tensor. For TM modes, the equations are decoupled and the field equations inside the core region ($r \leqslant R$) become

$$\frac{\partial^2 H_\phi}{\partial r^2}+\left(\frac{1}{r}+\frac{1}{\varepsilon_{rr}}\frac{\partial\varepsilon_{rr}}{\partial r}-2\mathrm{i}\beta\frac{\varepsilon_{rz}}{\varepsilon_{rr}}\right)\frac{\partial H_\phi}{\partial r}$$

$$+\left[\omega^2\mu_0\varepsilon_0\frac{\varepsilon_\parallel\varepsilon_\perp}{\varepsilon_{rr}}-\beta^2\frac{\varepsilon_{zz}}{\varepsilon_{rr}}-\frac{1}{r^2}-\frac{\mathrm{i}\beta}{\varepsilon_{rr}}\left(\frac{\partial\varepsilon_{rz}}{\partial r}+\frac{\varepsilon_{rz}}{r}\right)+\frac{1}{r\varepsilon_{rr}}\frac{\partial\varepsilon_{rr}}{\partial r}\right]H_\phi=0, \quad (6)$$

$$E_r=\frac{1}{\mathrm{i}\omega\varepsilon_0\varepsilon_\parallel\varepsilon_\perp}\left[\mathrm{i}\beta\varepsilon_{zz}H_\phi-\varepsilon_{rz}\left(\frac{\partial H_\phi}{\partial r}+\frac{H_\phi}{r}\right)\right], \quad (7)$$

$$E_z=\frac{\mathrm{i}}{\omega\varepsilon_0\varepsilon_\parallel\varepsilon_\perp}\left[\mathrm{i}\beta\varepsilon_{rz}H_\phi-\varepsilon_{rr}\left(\frac{\partial H_\phi}{\partial r}+\frac{H_\phi}{r}\right)\right]. \quad (8)$$

The boundary conditions at the core–cladding interface $r=R$ are

$$H_\phi^{\mathrm{in}}=H_\phi^{\mathrm{out}}, \quad (9)$$

$$\frac{1}{\varepsilon_\perp}\left(\frac{\partial H_\phi^{\mathrm{in}}}{\partial r}+\frac{H_\phi^{\mathrm{in}}}{r}\right)=\frac{1}{\varepsilon_{\mathrm{c}}}\left(\frac{\partial H_\phi^{\mathrm{out}}}{\partial r}+\frac{H_\phi^{\mathrm{out}}}{r}\right), \quad (10)$$

where H_ϕ^{in} and H_ϕ^{out} are fields in the core and cladding, and at $r=0$, $H_\phi(0)=0$.

It is possible to transform equation (6) into real self-adjoint form. If

$$H_\phi=f(z)\exp\left(\mathrm{i}\beta R\int\frac{\varepsilon_{rz}}{\varepsilon_{rr}}dz\right),$$

equation (6) becomes

$$f''+\left[\frac{2}{z}+\left(\ln\frac{\varepsilon_{rr}}{z}\right)'\right]f'+\left[\frac{1}{z}\left(\ln\frac{\varepsilon_{rr}}{z}\right)'+\frac{\varepsilon_\parallel\varepsilon_\perp}{\varepsilon_{rr}^2}(\omega^2\mu_0\varepsilon_0\varepsilon_{rr}-\beta^2)R^2\right]f=0, \quad (11)$$

where primes denote differentiation with respect to z. It is interesting to note that the phase of H_ϕ is a function of distance from the axis. Boundary conditions are $f(0)=0$, and, at the core–cladding interface at $z=1$

$$f^{\mathrm{in}}=f^{\mathrm{out}}, \quad (12)$$

$$\frac{\varepsilon_{rr}}{\varepsilon_\parallel\varepsilon_\perp}\left(f'^{\mathrm{in}}+\frac{f^{\mathrm{in}}}{z}\right)=\frac{1}{\varepsilon_{\mathrm{c}}}\left(f'^{\mathrm{out}}+\frac{f^{\mathrm{out}}}{z}\right), \quad (13)$$

where f^{in} and f^{out} are the magnitude of fields in the core and cladding.

This is a non-linear eigenvalue problem, where the propagation constant β plays the role of the eigenvalue. The field distributions and β can be obtained numerically by a simple adaptation of the shooting method [9]. Once the field distributions are known, then equation (5) can be solved numerically for $\theta(r)$. The scheme we used is as follows: we start with the director configuration which minimizes the free energy in the absence of fields, and use the corresponding dielectric tensor to obtain the field distributions for the propagating modes from Maxwell's equations. Next we calculate the change in the dielectric tensor due to torques resulting from these optical fields, and solve for the field distribution again. Iterating this process until we obtain the steady state director configuration, we determine the field distributions and the propagation constant β.

Figure 3 shows the magnetic field distribution for different values of the power P/P_0. The peak of the magnetic field tends to shift toward the centre of the waveguide as the power increases. Figures 4 and 5 show the distributions of the magnitude of fields D_r and E_z. Since E_r is discontinuous across the surface of the cylinder, D_r is shown on the graph. The peak value of E_r is about 5 times greater than that of E_z. The director configurations for different values of the optical power P/P_0 are shown in figure 6.

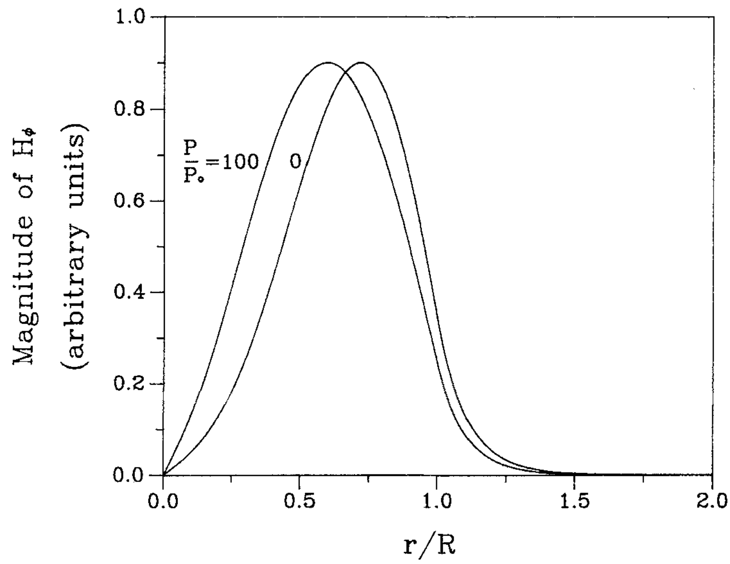


Figure 3. The magnitude of the magnetic field of the TM_{01} mode for different values of power P/P_0.

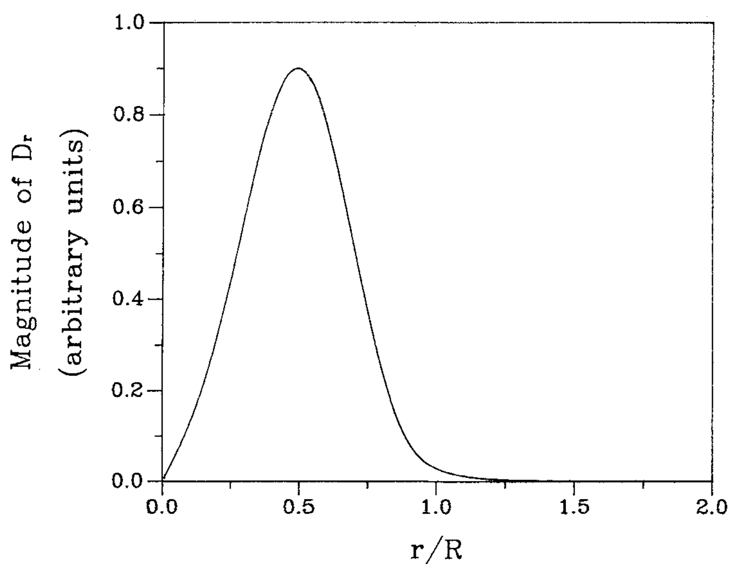


Figure 4. The magnitude of the electric field D_r of the TM_{01} mode.

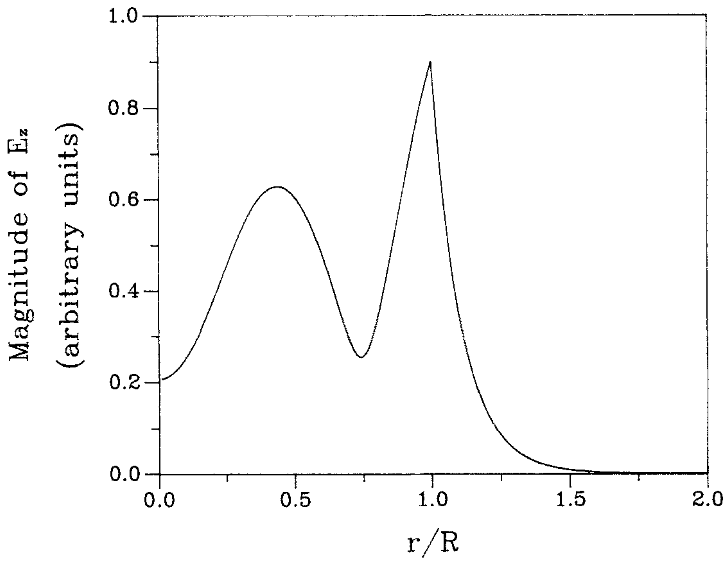


Figure 5. The magnitude of the electric field E_z of the TM_{01} mode.

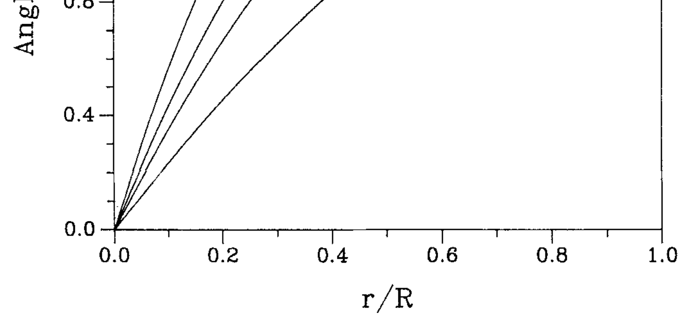


Figure 6. The director configuration for different values of power P/P_0.

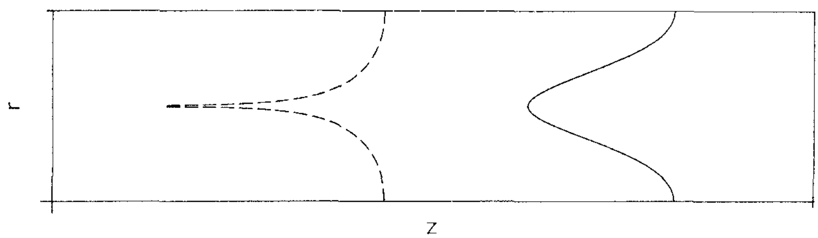


Figure 7. The director configuration of the liquid crystal core (dashed line) and the wavefront of H_ϕ in the waveguide (solid line).

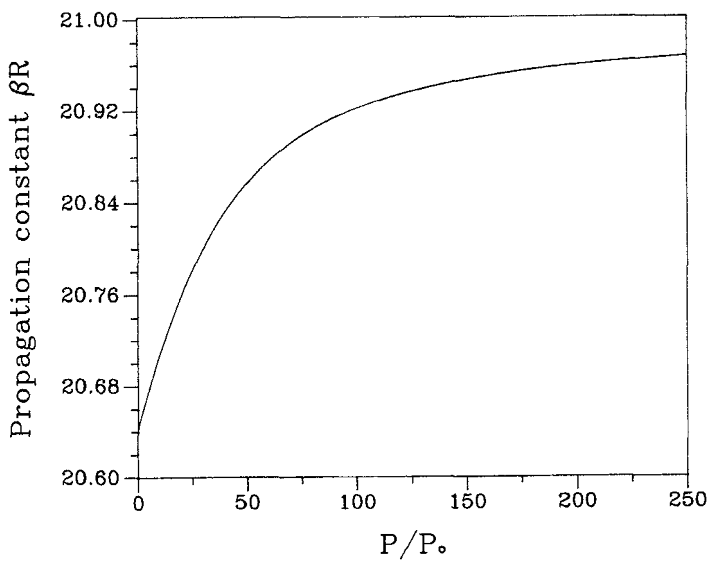


Figure 8. The propagation constant as a function of power P/P_0 for TM_{01} mode.

Away from the axis, $E_r > E_z$, and the director tends to align in the radial direction as the power increases. The director configuration and the wavefront of H_ϕ in the waveguide are shown in figure 7. Due to the anisotropy of the liquid crystal core, the ray and wave normals are not parallel. Figure 8 shows the propagation constant β as a function of power P/P_0 for the TM_{01} mode.

3. Non-linear effect of the TE modes

With the Landau–de Gennes formalism [11], there can be two distinct configurational changes due to the optical field which satisfy the boundary conditions [12]. One corresponds to a rotation of the director, i.e. the eigenvalues of $Q_{\alpha\beta}$ remain essentially the same while the orientation of the eigenframe varies with position. In the other the eigenvalues of $Q_{\alpha\beta}$ vary in space, while the eigenvectors are eventually fixed. We only consider in the latter a biaxial configuration. The reason for this is that the optical fields have three non-vanishing components E_ϕ, H_r and H_z for the TE modes. The electric field E^ϕ may induce a rotation of the direction out of the t–t plane; this will introduce new non-zero off-diagonal elements in the dielectric tensor. In this case, the field equations cannot be decoupled, and our approach is no longer easily viable. Therefore we restrict ourselves to the biaxial configuration.

The Landau–de Gennes free energy density is formed from scalar invariants of $Q_{\alpha\beta}$ and its derivatives. In the presence of fields, the free energy up to sixth-order terms in the one elastic constant approximation, is

$$F=\int\{\tfrac{1}{2}A\operatorname{tr}Q^2-\tfrac{1}{3}B\operatorname{tr}Q^3+\tfrac{1}{4}C(\operatorname{tr}Q^2)^2+\tfrac{1}{5}D\operatorname{tr}Q^2\operatorname{tr}Q^3+\tfrac{1}{6}E(\operatorname{tr}Q^2)^3$$
$$+\tfrac{1}{6}G(\operatorname{tr}Q^3)^2+\tfrac{1}{2}LQ_{\alpha\beta,\gamma}Q_{\alpha\beta,\gamma}-\tfrac{1}{2}wQ_{\alpha\beta}E_\alpha E_\beta\}d^3\mathbf{r},\tag{14}$$

where $A=a_0(T-T_c^*)$, T is the temperature, a_0, T_c^*, B, C, D, E, F, G, L, w are material constants; and $w=2(\varepsilon_{\parallel}-\varepsilon_{\perp})/3S$, where S is the orientational scalar order parameter in a uniaxial nematic phase. $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ are assumed to be a linear function of S, and w is a constant. Also, $\operatorname{tr}Q^2=Q_{\alpha\beta}Q_{\alpha\beta}$, $\operatorname{tr}Q^3=Q_{\alpha\beta}Q_{\beta\gamma}Q_{\gamma\beta}$ and $Q_{\alpha\beta,\gamma}=\partial Q_{\alpha\beta}/\partial x_\gamma$; E_α is the component of the electric field. For simplicity we have used the one elastic constant approximation.

We use cylindrical coordinates (r,ϕ,z) in a lab-fixed frame, and assume that one eigenvector of the order parameter tensor $Q_{\alpha\beta}$ is always parallel to the ϕ axis, and that $Q_{\alpha\beta}$ is a function only of r. $Q_{\alpha\beta}$ can then be expressed as

$$Q=\begin{bmatrix}\tfrac{1}{4}(S+P)-\tfrac{1}{4}(3S-P)\cos 2\theta & 0 & \tfrac{1}{4}(3S-P)\sin 2\theta\\ 0 & -\tfrac{1}{2}(S+P) & 0\\ \tfrac{1}{4}(3S-P)\sin 2\theta & 0 & \tfrac{1}{4}(S+P)+\tfrac{1}{4}(3S-P)\cos 2\theta\end{bmatrix},\tag{15}$$

where S and P are the scalar orientational order parameters: $S=\langle\tfrac{1}{2}(\mathbf{u}\cdot\mathbf{i})^2-1\rangle$ and $P=\langle\tfrac{3}{2}[(\mathbf{u}\cdot\mathbf{j})^2-(\mathbf{u}\cdot\mathbf{k})^2]\rangle$ where $\mathbf{u}$ is a unit vector along the symmetry axis of a molecule, $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are the eigenvectors of $Q_{\alpha\beta}$ and $\langle\quad\rangle$ denotes the ensemble average. In this representation, $\mathbf{i}$ and $\mathbf{k}$ are parallel to the r and z axes, respectively, at $r=0$. For $r\neq 0$, the eigenframe rotates through an angle $\theta(r)$ about the $\mathbf{j}$ axis. Thus $\theta(0)=0$, and $\theta(R)=\pi/2$. $\mathbf{j}$ is assumed to remain parallel to the ϕ axis throughout.

For TE modes, in this representation, the free energy can be written in cylindrical coordinates

$$\mathscr{F}=\int z\Big\{\tfrac{1}{4}a(3S^2+P^2)-\tfrac{1}{4}bS(S^2-P^2)+\tfrac{1}{16}(3S^2+P^2)^2+\tfrac{3}{40}dS(S^2-P^2)(3S^2+P^2)$$
$$+\tfrac{1}{48}e(3S^2+P^2)^3+\tfrac{3}{32}gS^2(S^2-P^2)^2+\frac{1}{8}\frac{1}{z^2}(9S^2+5P^2+6SP)$$
$$-\frac{3}{8}\frac{1}{z^2}(3S-P)(S+P)\cos 2\theta+\frac{1}{4}\left[3\left(\frac{\partial S}{\partial z}\right)^2+\left(\frac{\partial P}{\partial z}\right)^2\right]+\tfrac{1}{4}(3S-P)^2\left(\frac{\partial\theta}{\partial z}\right)^2$$
$$+\tfrac{1}{4}(S+P)\frac{w}{C}E_\phi^2\Big\}dz,\tag{16}$$

where the dimensionless free energy $\mathscr{F}=F/(2A'HC)$, A' is the cross-sectional area of the cylinder, $\mathbf{H}$ is the length of the cylinder; and $a=A/C$, $b=B/C$, $d=D/C$, $e=E/C$, $g=G/C$, $l=L/(R^2C)$, and

$$\frac{w}{C}E_\phi^2=\frac{1}{3\pi}\frac{E_\phi^2}{\int_0^1 H_r^* E_\phi z\,dz}\left(\frac{P}{P_0}\right),\quad \text{where } P_0=\frac{cL}{(\Delta\varepsilon/S)},$$

and c is the speed of light.

Configuration which minimize the free energy $\mathscr{F}$ correspond to solutions of the Euler–Lagrange equations $\delta f/\delta S=\delta f/\delta P=\delta f/\delta\theta=0$, where f is the integrand in equation (16). These give rise to three coupled non-linear differential equations. We use a relaxation method [12] to solve these equations numerically. In cylindrical coordinates, the Landau–Khalatnikov equations are

$$z\gamma_i\frac{\partial x}{\partial t}=-\frac{\delta f}{\delta x}=-\frac{\partial f}{\partial x}+\frac{d}{dz}\frac{\partial f}{\partial(dx/dz)},\tag{17}$$

where x represents S, P or θ, γ_i $(i=1,2,3)$ is a viscosity coefficient and t is time. Explicitly, these are

$$\begin{aligned}\gamma_1\frac{\partial S}{\partial t}=-\Big\{&\tfrac{3}{2}aS-\tfrac{1}{4}b(3S^2-P^2)+\tfrac{3}{4}S(3S^2+P^2)+\tfrac{3}{40}d[(3S^2-P^2)(3S^2+P^2)\\&+6S^2(S^2-P^2)]+\tfrac{3}{8}eS(3S^2+P^2)^2+\tfrac{3}{16}gS(S^2-P^2)(3S^2-P^2)\\&+\frac{3}{2}\frac{1}{z^2}(3S+P)\sin^2\theta+\tfrac{3}{2}l(3S-P)\left(\frac{\partial\theta}{\partial z}\right)^2\\&+\frac{1}{4}\frac{w}{C}E_\phi^2\Big\}+\frac{3}{2}l\left\{\frac{1}{z}\frac{\partial S}{\partial z}+\frac{\partial^2 S}{\partial z^2}\right\}\end{aligned}\tag{18}$$

$$\begin{aligned}\gamma_2\frac{\partial P}{\partial t}=-\Big\{&\tfrac{1}{2}aP+\tfrac{1}{2}bSP+\tfrac{1}{4}P(3S^2+P^2)-\tfrac{3}{10}dSP(S^2+P^2)\\&+\tfrac{1}{8}eP(3S^2+P^2)^2-\tfrac{3}{8}gS^2P(S^2-P^2)\\&+\frac{1}{4}\frac{l}{z^2}(5P+3S)-\frac{3}{4}\frac{l}{z^2}(S-P)\cos 2\theta-\tfrac{1}{2}l(3S-P)\left(\frac{\partial\theta}{\partial z}\right)^2\\&+\frac{1}{4}\frac{w}{C}E_\phi^2\Big\}+\frac{1}{2}l\left\{\frac{1}{z}\frac{\partial P}{\partial z}+\frac{\partial^2 P}{\partial z^2}\right\}\end{aligned}\tag{19}$$

$$\gamma_3\frac{\partial\theta}{\partial t}=-\frac{3}{4}\frac{l}{z^2}(3S-P)(S+P)\sin 2\theta+\tfrac{1}{2}l(3S-P)^2\left\{\frac{1}{z}\frac{\partial\theta}{\partial z}+\frac{\partial^2\theta}{\partial z^2}\right\}.\tag{20}$$

At the boundaries, θ is fixed, and we assume S can vary freely. $P=0$ at $r=0$ due to the cylindrical symmetry, and P can vary freely at $r=R$. This gives

$$\left.\begin{aligned}\theta&=0,\\ \frac{\partial S}{\partial z}&=0,\\ P&=0,\end{aligned}\right\}\tag{21}$$

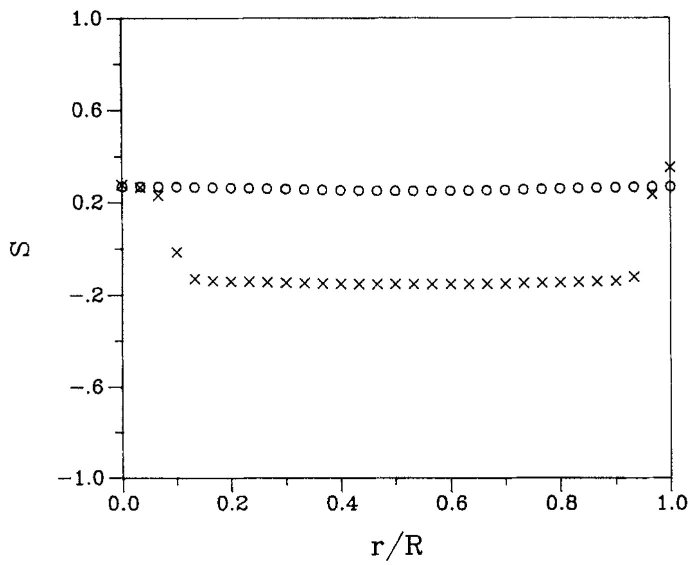


Figure 9. The order parameter S as a function of position for different values of power P/P_0. ○, $P/P_0 = 5 \times 10^5$; ×, $P/P_0 = 10 \times 10^5$.

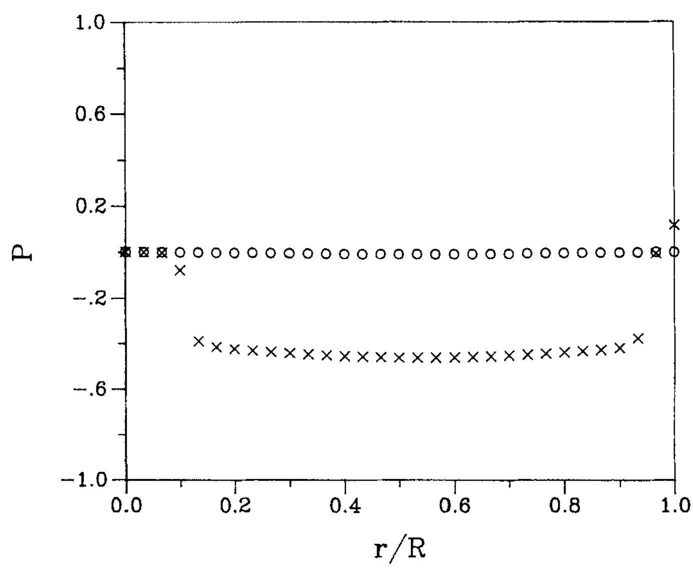


Figure 10. The order parameter P as a function of position for different values of power P/P_0. ○, $P/P_0 = 5 \times 10^5$; ×, $P/P_0 = 10 \times 10^5$.

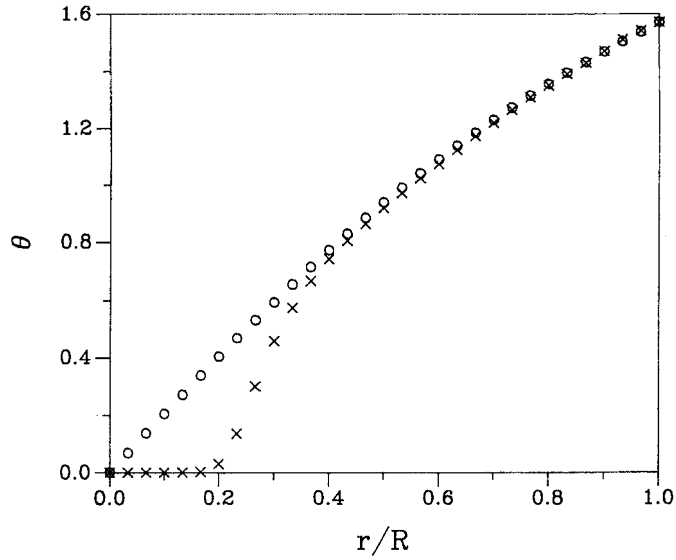


Figure 11. The director configuration as a function of position for different values of power P/P_0. ○, $P/P_0 = 5 \times 10^5$; ×, $P/P_0 = 10 \times 10^5$.

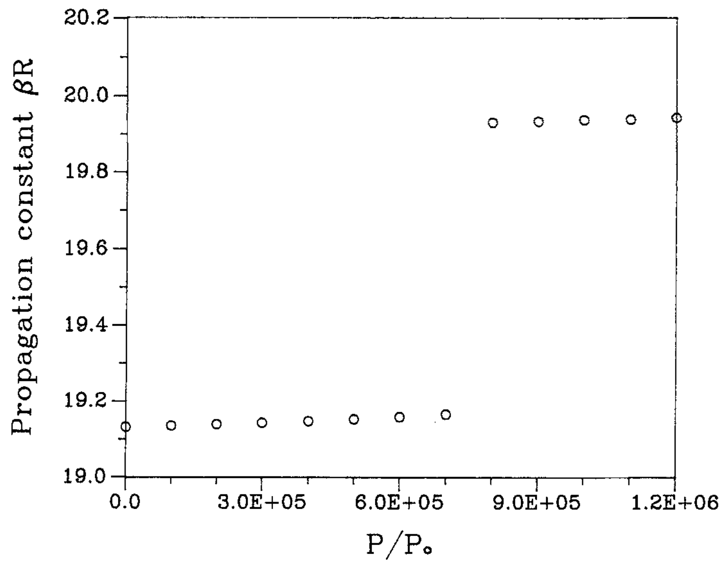


Figure 12. The propagation constant as a function of power P/P_0 for TE_{01} mode.

at $z=0$, and

$$\left.\begin{aligned} \theta &= \frac{\pi}{2}, \\ \frac{\partial S}{\partial z} &= 0, \\ \frac{\partial P}{\partial z} &= 0, \end{aligned}\right\} \tag{22}$$

at $z=1$.

If E_ϕ is known, the above equations can be solved using a standard relaxation method. For TE modes, the component of the electric field E_ϕ is the solution of the wave equation

$$\frac{\partial^2 E_\phi}{\partial r^2}+\frac{1}{r}\frac{\partial E_\phi}{\partial r}+\left(\omega^2\mu_0\varepsilon_0\varepsilon_{\phi\phi}-\beta^2-\frac{1}{r^2}\right)E_\phi=0. \tag{23}$$

In the biaxial configuration, the dielectric tensor has the form $\varepsilon_{\alpha\beta}=\bar{\varepsilon}\delta_{\alpha\beta}+wQ_{\alpha\beta}$, where $\bar{\varepsilon}=(\varepsilon_\parallel+2\varepsilon_\perp/3)$, and therefore $\varepsilon_{\phi\phi}=\bar{\varepsilon}-\frac{1}{2}w(S+P)$.

These equations are discretized and solved iteratively. We start with a set of initial values S, P and θ, and solve equation (23) for the field E_ϕ using the shooting method [9]. We then use finite differences to solve equations (18) to (20) for S, P and θ. Then we use the new values to solve equation (23) for E_ϕ again. This process is repeated until convergence.

Results on the distributions of S, P and θ for different optical powers inside the waveguide are shown in figures 9, 10 and 11. If the power is low, the biaxiality is very small, and the director configuration is close to the one described by the Frank free energy. As the power is increased, at a critical value ($P_c/P_0\simeq 8\times 10^5$), the average biaxiality suddenly increases. The order parameters S and P remain positive near the axis and the surface, but are negative elsewhere. This indicates that the molecules are effectively aligned with the field E_ϕ everywhere except at the boundaries. Figure 12 shows the propagation constant β as a function of power P/P_0 for the TE_{01} mode. This suggests that a first order transition occurs at the critical power.

4. Conclusion

We have studied the dependence of the propagation constant β on optical power in a cylindrical liquid crystal waveguide for both TE and TM modes. By exerting torques, the propagating fields change the configuration and the dielectric tensor of the liquid crystal core. For the TM and TE modes, by using an iterative numerical scheme, we have determined the steady state configuration, field distributions and propagation constant β. Because of the non-local response, β is a function of optical power rather than intensity. In the case of the TM mode, the propagation constant increases continuously with the optical power. In the case of the TE mode, a first order transition occurs at some critical power. We have considered only the transition to the biaxial configuration here; it is possible that another configuration is realized at lower power levels.

This research was supported in part by the National Science Foundation Science and Technology Center ALCOM DMR89-20147, and by the Defense Advanced Research Projects Agency through US Army contract DAAB07-88-C-F421 from the Center for Night Vision and Electro-optics.

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